Problem 6.17

$$x[n] = 5 + 20\cos\left(\tfrac{\pi}{2}n + \tfrac{\pi}{4}\right) + 10\delta[n-3]$$

Need $\mathcal{H}(0)$ DEPENDS on $\mathcal{H}(\pi/2)$ Need impulse response $h[n]$

$$\mathcal{H}(0) = (1-j)(1-(-j))(1+1)$$
$$= (1-j)(1+j)2 = 2\cdot 2 = 4$$

$$\mathcal{H}(\pi/2) = (1-je^{-j\pi/2})(1+je^{-j\pi/2})(1+e^{-j\pi/2})$$
$$= (1-j(-j))(1+j(-j))(1-j)$$
$$= (1-1)(1+1)(1-j) = 0$$

To find $h[n]$, multiply out $\mathcal{H}(\hat{\omega})$

$$\mathcal{H}(\hat{\omega}) = (1 - je^{-j\hat{\omega}} + je^{-j\hat{\omega}} + e^{-j2\hat{\omega}})(1+e^{-j\hat{\omega}})$$
$$= (1+e^{-j2\hat{\omega}})(1+e^{-j\hat{\omega}})$$
$$= 1 + e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} + e^{-j3\hat{\omega}}$$

$$\Rightarrow h[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$$

Finally,

$$y[n] = 5(4) + 0 + 10h[n-3]$$
$$= 20 + 10\delta[n-3] + 10\delta[n-4] + 10\delta[n-5] + 10\delta[n-6]$$

Problem 6.18

(a) $\mathcal{H}(\hat{\omega}) = \mathcal{H}_1(\hat{\omega})\mathcal{H}_2(\hat{\omega})$

$$\mathcal{H}_2(\hat{\omega}) = 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}}$$

$$\mathcal{H}_1(\hat{\omega}) = 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

Multiply:

$$\mathcal{H}(\hat{\omega}) = 1 - e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}} + 2e^{-j\hat{\omega}} - 2e^{-j2\hat{\omega}} + 2e^{-j3\hat{\omega}} - 2e^{-j4\hat{\omega}} + e^{-j2\hat{\omega}} - e^{-j3\hat{\omega}} + e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}}$$

$$\mathcal{H}(\hat{\omega}) = 1 + e^{-j\hat{\omega}} - e^{-j4\hat{\omega}} - e^{-j5\hat{\omega}}$$

MANY TERMS CANCEL OUT

(b) $h[n] = \delta[n] + \delta[n-1] - \delta[n-4] - \delta[n-5]$

(c) The polynomial coefficients of $\mathcal{H}(\hat{\omega})$ define $\{b_k\}$ as $\{1, 1, 0, 0, -1, -1\}$. Use $\{b_k\}$ as filter coefficients:

$$y[n] = x[n] + x[n-1] - x[n-4] - x[n-5]$$

Problem 7.4

(a) use filter coeffs: $H(z) = \frac{1}{3} + \frac{1}{3}z^{-1} + \frac{1}{3}z^{-2}$

(b) Use positive powers to extract poles and zeros

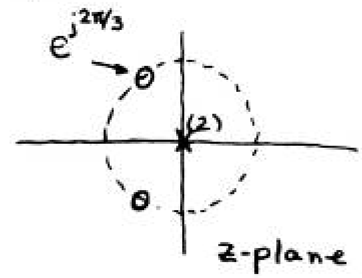
$$H(z) = \frac{1}{z^2}\left(\frac{1}{3}z^2 + \frac{1}{3}z + \frac{1}{3}\right)$$

TWO POLES AT $z=0$

zeros at

$$z = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

zeros: $1e^{\pm j2\pi/3}$



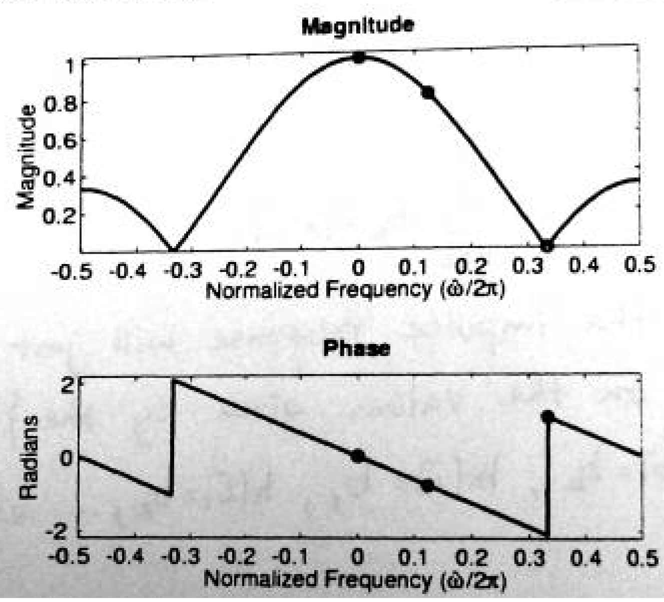
(c) $\mathcal{H}(\hat{\omega}) = H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}}$

$$= \frac{1}{3} + \frac{1}{3}e^{-j\hat{\omega}} + \frac{1}{3}e^{-j2\hat{\omega}} = \frac{1}{3}e^{-j\hat{\omega}}\left(e^{j\hat{\omega}} + 1 + e^{-j\hat{\omega}}\right)$$

$$= e^{-j\hat{\omega}}\left(\frac{1+2\cos\hat{\omega}}{3}\right)$$

$$\boxed{\text{ANOTHER FORMULA: } \mathcal{H}(\hat{\omega}) = e^{-j\hat{\omega}}\left(\frac{\sin(3\hat{\omega}/2)}{3\sin(\hat{\omega}/2)}\right)}$$

(d) use MATLAB



Problem 7.4 (more)

(e) Use Linearity & Frequency response at $\hat{\omega}=0$, $\hat{\omega}=\pi/4$ and $\hat{\omega}=2\pi/3$. These are marked on the plots of the frequency response.

$$y[n] = 4\mathcal{H}(0) + |\mathcal{H}(\pi/4)| \cos(\tfrac{\pi}{4}n - \tfrac{\pi}{4} + \angle\mathcal{H}(\pi/4)) - 3\underbrace{|\mathcal{H}(2\pi/3)|}_{=0} \cos(\tfrac{2\pi}{3}n + \angle\mathcal{H}(2\pi/3))$$

$$\mathcal{H}(0) = \tfrac{1}{3} + \tfrac{1}{3} + \tfrac{1}{3} = 1$$

$$\mathcal{H}(\pi/4) = e^{-j\pi/4}(1 + 2\sqrt{2}/2)/3 = \frac{1+\sqrt{2}}{3} e^{-j\pi/4} = 0.8047 e^{-j\pi/4}$$

$\mathcal{H}(2\pi/3) = 0$ because $H(z)=0$ at $z = e^{\pm j2\pi/3}$

$$\therefore\ y[n] = 4 + 0.8047 \cos(\tfrac{\pi}{4}n - \tfrac{\pi}{2})$$

Problem 7.9

(a) A 4-point moving average is

$$y[n] = \frac{1}{4}\left(x[n] + x[n-1] + x[n-2] + x[n-3]\right)$$

$$H_1(z) = H_2(z) = \frac{1}{4} + \frac{1}{4}z^{-1} + \frac{1}{4}z^{-2} + \frac{1}{4}z^{-3}$$

$$H(z) = H_1(z)H_2(z) = \frac{1}{16}\left(1 + z^{-1} + z^{-2} + z^{-3}\right)^2$$

(b) Find the poles and zeros of $H_1(z)$, then "double" them. Switch to positive powers of z.

$$H_1(z) = \frac{z^3+z^2+z+1}{4z^3}$$

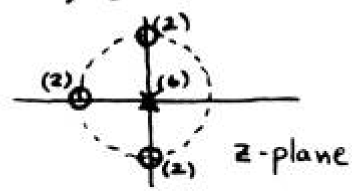
Numerator factors: $(z+1)(z^2+1)$

$= (z+1)(z+j)(z-j)$

3 poles at $z=0$

$\Rightarrow$ Zeros at $z = -1, -j, +j$

(c) For the freq. response



$$H(e^{j\hat{\omega}}) = H(z)\big|_{z=e^{j\hat{\omega}}}$$

$$H(e^{j\hat{\omega}}) = \frac{1}{16}\Big(\underbrace{1+e^{-j\hat{\omega}}+e^{-j2\hat{\omega}}+e^{-j3\hat{\omega}}}_{\text{This can be reduced to a Dirichlet form.}}\Big)^2$$

$$H(e^{j\hat{\omega}}) = \frac{1}{16}\left(\frac{\sin(4\hat{\omega}/2)}{\sin(\hat{\omega}/2)}\,e^{-j3\hat{\omega}/2}\right)^2$$

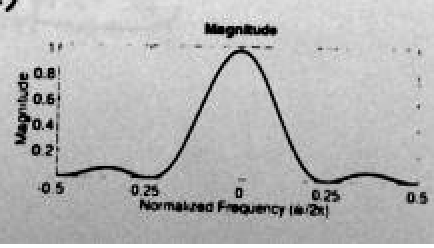
$$= e^{-j3\hat{\omega}}\left(\frac{\sin(2\hat{\omega})}{4\sin(\hat{\omega}/2)}\right)^2$$

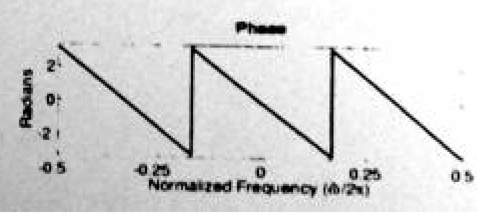
phase

Numerator is zero for $2\hat{\omega} = \pi k$ $\Rightarrow$ $\hat{\omega} = \pi k/2$

At $\hat{\omega}=0$, denominator is also zero

(d)





Problem 7.9 (more)

(e) $$H(z) = \frac{1}{16}\left(1 + z^{-1} + z^{-2} + z^{-3}\right)^2$$

$$= \frac{1}{16}\left(1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 3z^{-4} + 2z^{-5} + z^{-6}\right)$$

Invert term by term

$$h[n] = \frac{1}{16}\delta[n] + \frac{1}{8}\delta[n-1] + \frac{3}{16}\delta[n-2] + \frac{1}{4}\delta[n-3] + \frac{3}{16}\delta[n-4]$$

$$+ \frac{1}{8}\delta[n-5] + \frac{1}{16}\delta[n-6]$$