Practice Problems

3.60, 3.64, 3.72
4.3, 4.8, 4.9, 4.23, 4.37, 4.38

P3.60* (a) $L_{eq} = 1 + \dfrac{1}{1/6 + 1/(1+2)} = 3\text{ H}$

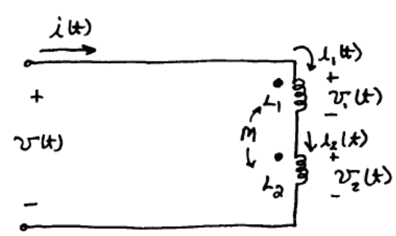
(b) 9 H in parallel with 18 H is equivalent to 6H. Also, 20 H in parallel with 5 H is equivalent to 4 H. Finally, we have $L_{eq} = \dfrac{1}{1/15 + 1/(6+4)} = 6\text{ H}$

P3.64 Ordinarily, negative inductance is not practical. Thus, adding inductance in series always increases the equivalent inductance. However, placing inductance in parallel results in smaller inductance. Thus, we need to consider a parallel inductance such that

$$\frac{1}{1/L + 1/4} = 3$$

Solving, we find that $L = 12$ H.

P3.72* (a)



As in Figure 3.23a, we can write

$$v_1(t) = L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt}$$

$$v_2(t) = M \frac{di_1(t)}{dt} + L_2 \frac{di_2(t)}{dt}$$

However, for the circuit at hand, we have $i(t) = i_1(t) = i_2(t)$.
Thus,

$$v_1(t) = (L_1 + M)\frac{di(t)}{dt}$$

$$v_2(t) = (L_2 + M)\frac{di(t)}{dt}$$

Also, we have $v(t) = v_1(t) + v_2(t)$.

Substituting, we obtain $v(t) = (L_1 + 2M + L_2)\frac{di(t)}{dt}$.

Thus, we can write $v(t) = L_{eq}\frac{di(t)}{dt}$, in which

$L_{eq} = L_1 + 2M + L_2$.

(b) Similarly, for the dot at the bottom end of L_2, we have

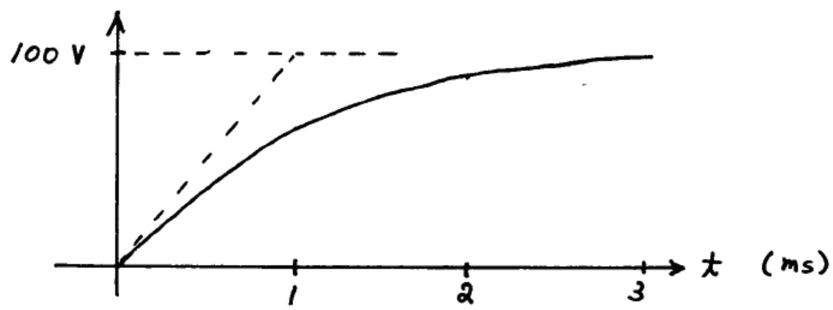
$L_{eq} = L_1 - 2M + L_2$

P4.3 The solution is of the form given in Equation 4.19:

$$v_C(t) = V_s - V_s \exp(-t/RC)$$
$$RC = 10^5 \times 0.01 \times 10^{-6} = 1 \text{ ms}$$

Thus, we have

$$v_C(t) = 100 - 100\exp(-t/10^{-3})$$



P4.8 Equation 4.8 gives the expression for the voltage across a capacitance discharging through a resistance:

$$v_C(t) = V_i \exp(-t/RC)$$

After one-half-life, we have $v_C(t_{half}) = \frac{V_i}{2}$ and $\frac{V_i}{2} = V_i \exp(-t_{half}/RC)$.

Dividing by V_i and taking the natural logarithm of both sides, we have

$$-\ln(2) = -t_{half}/RC$$

Solving, we obtain

$$t_{half} = RC\ln(2) = 0.6931RC = 0.6931\tau$$

P4.9 Prior to $t = 0$, we have $v(t) = 0$ because the switch is closed. After $t = 0$, we can write the following KCL equation at the top node of the circuit:

$$\frac{v(t)}{R} + C\frac{dv(t)}{dt} = 1\text{ mA}$$

Multiplying both sides by R and substituting values, we have

$$0.01\frac{dv(t)}{dt} + v(t) = 10 \qquad (1)$$

The solution is of the form

$$v(t) = K_1 + K_2\exp(-t/RC) = K_1 + K_2\exp(-100t) \qquad (2)$$

Substituting Equation (2) into Equation (1), we eventually obtain

$$K_1 = 10$$

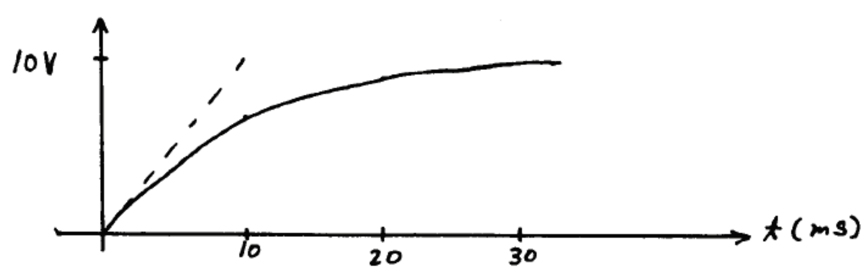
The voltage across the capacitance cannot change instantaneously, so we have

$$v(0+) = v(0-) = 0$$

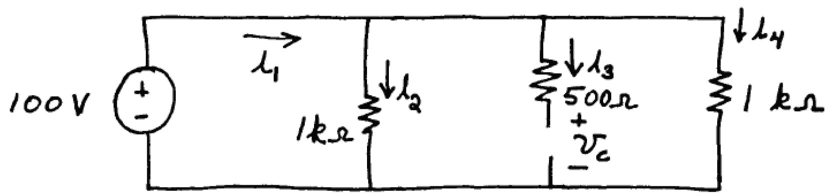
$$v(0+) = 0 = K_1 + K_2$$

Thus, $K_2 = -K_1 = -10$, and the solution is

$$v(t) = 10 - 10\exp(-100t) \text{ for } t > 0$$



P4.23 In steady state with a dc source, the inductance acts as a short circuit and the capacitance acts as an open circuit. The equivalent circuit is:



$$i_4 = (100\text{ V})/(1\text{ k}\Omega) = 100\text{ mA}$$
$$i_3 = 0$$
$$i_2 = (100\text{ V})/(1\text{ k}\Omega) = 100\text{ mA}$$
$$i_1 = i_2 + i_3 + i_4 = 200\text{ mA}$$
$$v_C = 100\text{ V}$$

P4.37 In steady state, the inductor acts as a short circuit. With the switch open, the steady-state current is $(100\text{ V})/(100\ \Omega) = 1\,A$. With the switch closed, the current eventually approaches $i(\infty) = (100\text{ V})/(25\ \Omega) = 4\ A$. For $t > 0$, the current has the form

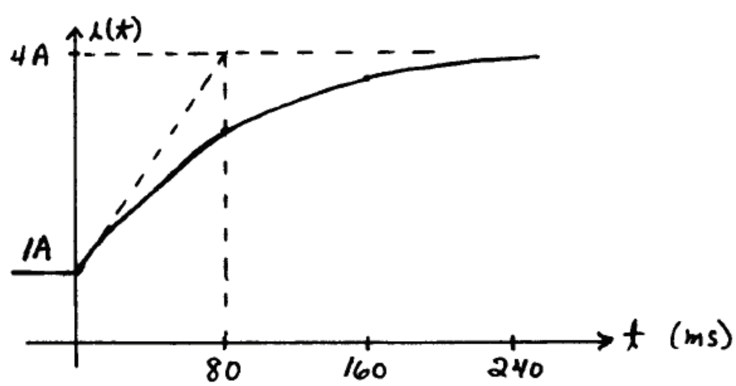
$$i(t) = K_1 + K_2 \exp(-Rt/L)$$

where $R = 25\ \Omega$, because that is the resistance with the switch closed. Now, we have

$$i(0+) = i(0-) = 1 = K_1 + K_2$$
$$i(\infty) = 4 = K_1$$

Thus, we have $K_2 = -3$. The current is

$$\begin{aligned} i(t) &= 1 & t < 0\text{ (switch open)} \\ &= 4 - 3\exp(-12.5t) & t \geq 0\text{ (switch closed)} \end{aligned}$$



P4.38 Before the switch closes, 1 A of current circulates through the source and the two 10-Ω resistors. Immediately after the switch closes, the inductor current remains 0 A, because infinite voltage is not possible in this circuit. (Because the inductor current is zero we can consider the inductor to be an open circuit at $t = 0+$.) Therefore, the current through the resistors is unchanged, and $i(0+) = 1$ A. In steady state, the inductor acts as a short circuit, and we have $i(\infty) = 2$ A. The Thévenin resistance seen by the inductor is 5 Ω because the two 10-Ω resistors are in parallel when we zero the source and look back into the circuit from the inductor terminals. Thus, the time constant is $\tau = L/R = 200$ ms. The general form of the solution is $i(t) = K_1 + K_2 \exp(-t/\tau)$. Using the initial and final values, we have $i(0+) = 1 = K_1 + K_2$ and $i(\infty) = 2 = K_1$ which yields $K_2 = -1$. Thus, the current is

$$\begin{aligned} i(t) &= 1 \quad t \le 0 \\ &= 2 - \exp(-t/\tau) \quad t \ge 0 \end{aligned}$$